

Version



**General Certificate of Education (A-level)
January 2013**

Mathematics

MFP4

(Specification 6360)

Further Pure 4

Final

Mark Scheme

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
✓ or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
-x EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q	Solution	Marks	Total	Comments
1	$\mathbf{n}_1 = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} \quad \mathbf{n}_2 = \begin{bmatrix} p \\ 3 \\ 0 \end{bmatrix}$ $\mathbf{n}_1 \cdot \mathbf{n}_2 = p+6$ $ \mathbf{n}_1 = \sqrt{1^2 + 2^2 + 2^2} = 3$ $ \mathbf{n}_2 = \sqrt{p^2 + 9}$ <p>Using $\mathbf{a} \cdot \mathbf{b} = \mathbf{a} \mathbf{b} \cos \theta$:</p> <p>'their' $p+6 = (3)(\sqrt{p^2+9})\left(\frac{2}{3}\right)$</p> $\Rightarrow (p+6)^2 = 4p^2 + 36$ $p^2 + 12p + 36 = 4p^2 + 36$ $0 = 3p(p-4)$ $p \neq 0 \Rightarrow p = 4$	<p>B1</p> <p>M1A1</p> <p>m1</p> <p>A1</p>	<p>5</p> <p>5</p>	<p>$\mathbf{n}_1 \cdot \mathbf{n}_2$ correct</p> <p>forming an equation using scalar product</p> <p>correctly forming and attempting to solve their quadratic equation</p> <p>$p = 4$ stated clearly (must reject $p=0$)</p>
Total			5	
2(a)	$\det \mathbf{A}^{-1} = -3 \Rightarrow \det \mathbf{A} = -\frac{1}{3}$	B1	1	
(b)	$\det(\mathbf{AB}) = 24 \Rightarrow \det \mathbf{B} = \frac{24}{\det \mathbf{A}} = -72$ $\text{Volume} = 20 \times 72 = 1440 \text{ cm}^3$	<p>M1</p> <p>A1F</p> <p>A1cso</p>	<p>3</p> <p>3</p>	<p>M1 for use of $\det(\mathbf{AB}) = \det \mathbf{A} \times \det \mathbf{B}$</p> <p>A1F ft their $\det \mathbf{A}$</p> <p>Must be positive</p>
Total			4	
3(a)	$(\mathbf{a} - 4\mathbf{b}) \times (\mathbf{a} + 3\mathbf{b}) = \mathbf{a} \times \mathbf{a} - 4\mathbf{b} \times \mathbf{a} + 3\mathbf{a} \times \mathbf{b} - 12\mathbf{b} \times \mathbf{b}$ $= -4\mathbf{b} \times \mathbf{a} + 3\mathbf{a} \times \mathbf{b}$ $= 7\mathbf{a} \times \mathbf{b}$	<p>M1</p> <p>A1</p> <p>A1cso</p>	<p>3</p>	<p>Three terms correct</p> <p>$\mathbf{b} \times \mathbf{b} = \mathbf{a} \times \mathbf{a} = \mathbf{0}$ — correct use</p> <p>$7\mathbf{a} \times \mathbf{b}$ or $-7\mathbf{b} \times \mathbf{a}$</p>
(b)	$\mathbf{a} \perp \mathbf{b} \Rightarrow \sin \theta = 1$ $\Rightarrow \mathbf{a} \times \mathbf{b} = \mathbf{a} \mathbf{b} $ $\Rightarrow (\mathbf{a} - 4\mathbf{b}) \times (\mathbf{a} + 3\mathbf{b}) = 7 \mathbf{a} \mathbf{b} $ $\lambda = 7$	<p>M1</p> <p>A1F</p>	<p>2</p>	<p>Use of $\sin \theta = 1$ to simplify</p> <p>Should match 'their' 7</p>
Total			5	

Q	Solution	Marks	Total	Comments
4(a)	$\mathbf{A}^2 = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$ $= \begin{bmatrix} -1 & -2 & -4 \\ 5 & 6 & 4 \\ 10 & 10 & 9 \end{bmatrix} \quad \begin{array}{l} p = -1 \\ q = 10 \end{array}$	B1 B1	2	p -value q -value
(b)	$\mathbf{A}^3 - 6\mathbf{A}^2 + 11\mathbf{A} - 6\mathbf{I} = \mathbf{0}$ multiply by \mathbf{A}^{-1} $(\mathbf{A}^3 - 6\mathbf{A}^2 + 11\mathbf{A} - 6\mathbf{I})\mathbf{A}^{-1} = (\mathbf{0})\mathbf{A}^{-1}$ $\mathbf{A}^3\mathbf{A}^{-1} - 6\mathbf{A}^2\mathbf{A}^{-1} + 11\mathbf{A}\mathbf{A}^{-1} - 6\mathbf{I}\mathbf{A}^{-1} = \mathbf{0}$ $\mathbf{A}^2 - 6\mathbf{A} + 11\mathbf{I} - 6\mathbf{A}^{-1} = \mathbf{0}$ $6\mathbf{A}^{-1} = \mathbf{A}^2 - 6\mathbf{A} + 11\mathbf{I}$ $\mathbf{A}^{-1} = \frac{1}{6}(\mathbf{A}^2 - 6\mathbf{A} + 11\mathbf{I})$	M1 A1	2	Multiplication by \mathbf{A}^{-1} AG
(c)	$\mathbf{A}^{-1} = \frac{1}{6} \begin{bmatrix} 4 & -2 & 2 \\ -1 & 5 & -2 \\ -2 & -2 & 2 \end{bmatrix} \quad \begin{array}{l} r = 4 \\ s = -2 \end{array}$	B1 B1	2	r -value s -value
(d)	$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{6} \begin{bmatrix} 4 & -2 & 2 \\ -1 & 5 & -2 \\ -2 & -2 & 2 \end{bmatrix} \begin{bmatrix} k \\ 5 \\ 7 \end{bmatrix}$ $= \frac{1}{6} \begin{bmatrix} 4k - 10 + 14 \\ -k + 25 - 14 \\ -2k - 10 + 14 \end{bmatrix} = \frac{1}{6} \begin{bmatrix} 4k + 4 \\ 11 - k \\ 4 - 2k \end{bmatrix}$ $x = \frac{2k + 2}{3}, y = \frac{11 - k}{6}, z = \frac{2 - k}{3}$	M1 A1 A1	3	use of $\mathbf{A}^{-1} \mathbf{v}$ – one row correct correct solution for one variable all correct CAO
Total			9	
(d)	alternative If solving equations by elimination, M1 A1 for correct solution for one variable, A1 all correct			

Q	Solution	Marks	Total	Comments
5(a)	$\begin{vmatrix} -2 & 1 & 2k \\ -1 & 1 & k+1 \\ 2 & k-1 & 1 \end{vmatrix} =$ $= -2 \begin{vmatrix} 1 & k+1 \\ k-1 & 1 \end{vmatrix} + 1 \begin{vmatrix} 1 & 2k \\ k-1 & 1 \end{vmatrix} + 2 \begin{vmatrix} 1 & 2k \\ 1 & k+1 \end{vmatrix}$ $= -2[1-(k+1)(k-1)] + [1-2k(k-1)] + 2[k+1-2k]$ $= -2[1-k^2+1] + [1-2k^2+2k] + 2[1-k]$ $= -4 + 2k^2 + 1 - 2k^2 + 2k + 2 - 2k$ $= -1$ <p><i>either all k's cancel or independent of k etc</i></p>	M1 A1 A1cso		correctly expanding by any row or column correct unsimplified expansion of 2×2 determinants -1 obtained
(b)	Identifying that $k = 2$ Value of determinant $\neq 0$ (or $= -1$ etc) therefore vectors are linearly independent	B1		$k = 2$
(c)(i)	Identifying that $k = 3$ Value of determinant $\neq 0$ (or $= -1$ etc) therefore equations are consistent	E1F B1	2	ft answer (a) if $0 \Rightarrow$ lin dep $k = 3$
(ii)	3 planes intersect in a unique point	E1F B1	2	ft answer (a) if $0 \Rightarrow$ inconsistent
	Total		9	
	<p>Alternative for (a):</p> $\begin{vmatrix} -2 & 1 & 2k \\ -1 & 1 & k+1 \\ 2 & k-1 & 1 \end{vmatrix} \quad \begin{array}{l} r_1 \rightarrow r_1 - 2r_2 \\ r_3 \rightarrow r_3 + 2r_2 \end{array}$ $= \begin{vmatrix} 0 & -1 & -2 \\ -1 & 1 & k+1 \\ 0 & k+1 & 2k+3 \end{vmatrix} \quad r_3 \rightarrow r_3 + (k+1)r_1$ $= \begin{vmatrix} 0 & -1 & -2 \\ -1 & 1 & k+1 \\ 0 & 0 & 1 \end{vmatrix} = -1$ <p><i>either all k's cancel or independent of k etc</i></p>	(M1) (A1) (A1) (E1)	(4)	correctly expanding by any row or column after row operations correct expansion unsimplified -1 obtained comment required

Q	Solution	Marks	Total	Comments
6(a)(i)	Reflection In (the plane) $z=0$ (or in the x - y plane)	M1 A1	2	Reflection stated for M1 Either version for A1
	(ii) Rotation About the y -axis through $\frac{\pi}{3}$ radians	M1 A1 B1	3	Rotation stated. y -axis (or 60°)
(b)	$T_2 T_1 = \begin{bmatrix} \frac{1}{2} & 0 & \frac{\sqrt{3}}{2} \\ 0 & 1 & 0 \\ -\frac{\sqrt{3}}{2} & 0 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & -\frac{\sqrt{3}}{2} \\ 0 & 1 & 0 \\ -\frac{\sqrt{3}}{2} & 0 & -\frac{1}{2} \end{bmatrix}$	M1A1	2	M1 correct order of matrices A1 fully correct [N.B. $\begin{bmatrix} \frac{1}{2} & 0 & \frac{\sqrt{3}}{2} \\ 0 & 1 & 0 \\ \frac{\sqrt{3}}{2} & 0 & -\frac{1}{2} \end{bmatrix}$ scores M0A0]
(c)(i)	For line of invariants points			
	$\begin{bmatrix} k & 2 & -1 \\ 1 & 1 & 1 \\ 3 & 4 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$	M1		Set up equations – uses $\mathbf{M} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$
	$\Rightarrow kx + 2y - z = x \Rightarrow (k-1)x + 2y - z = 0$ ①			
	$x + y + z = y \Rightarrow x + z = 0$ ②	A1		Two equations correct
	$3x + 4y + z = z \Rightarrow 3x + 4y = 0$ ③	A1		All three equations correct
	From ② $z = -x$	M1		Defines variables in terms of one letter
	From ③ $y = \frac{-3x}{4}$			or 2 components in $\begin{bmatrix} 4 \\ -3 \\ -4 \end{bmatrix}$ correct
Substitute in ① $(k-1)x - \frac{3}{2}x + x = 0$	A1		Substitution into other equations	
$x \left[k - 1 - \frac{3}{2} + 1 \right] = 0$			or $\begin{bmatrix} 4 \\ -3 \\ -4 \end{bmatrix}$ correct	
$x \left[k - \frac{3}{2} \right] = 0$				
$x \neq 0 \Rightarrow k = \frac{3}{2}$	A1		k -value obtained.	
(ii)	Line $x = \frac{-4}{3}y = -z$ $\frac{x}{4} = \frac{y}{-3} = \frac{z}{-4}$	B1cao	7	Or equivalent
Total			14	

Q	Solution	Marks	Total	Comments
	<p>Alternative to 6(c):</p> $\begin{bmatrix} k-1 & 2 & -1 & 0 \\ 1 & 0 & 1 & 0 \\ 3 & 4 & 0 & 0 \end{bmatrix}$ <p>$r_1 \rightarrow r_1 + r_2$</p> $\begin{bmatrix} k & 2 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 3 & 4 & 0 & 0 \end{bmatrix}$ <p>$r_1 \rightarrow r_1 - \frac{1}{2} r_3$</p> $\begin{bmatrix} k - \frac{3}{2} & 2 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 3 & 4 & 0 & 0 \end{bmatrix} \Rightarrow k = \frac{3}{2}$ <p>$v = \lambda \begin{pmatrix} 4 \\ -3 \\ -4 \end{pmatrix}$</p> $\frac{x}{4} = \frac{y}{-3} = \frac{z}{-4}$	<p>(M1)</p> <p>(A1)</p> <p>(A1)</p> <p>(A1)</p> <p>(M1A1)</p> <p>(B1cao)</p>	<p>(7)</p>	<p>Substitute and set up</p> <p>Row operation</p> <p>Row operation</p> <p>k-value obtained</p> <p>M1 obtains v in terms of single vector. A1 correct</p> <p>Correct form</p>

Q	Solution	Marks	Total	Comments
7(a)	$\begin{bmatrix} -a & 0 & a \\ 0 & 6 & 0 \\ a & 0 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 6 \\ 0 \end{bmatrix}$ $\Rightarrow \lambda_1 = 6$	M1 A1	2	
(b)	$\begin{bmatrix} -a & 0 & a \\ 0 & 6 & 0 \\ a & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} a \\ 0 \\ a+4 \end{bmatrix}$ $\begin{bmatrix} a \\ 0 \\ a+4 \end{bmatrix} = \begin{bmatrix} \lambda_2 \\ 0 \\ 2\lambda_2 \end{bmatrix}$ <p>i component $\Rightarrow a = \lambda_2 \otimes$ k component $\Rightarrow a + 4 = 2\lambda_2$ using \otimes, $a + 4 = 2a$ $4 = a$</p>	M1 m1 A1	3	Eliminating λ_2 Value of a obtained
(c)	<p>Let $\mathbf{v}_3 = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$</p> $\begin{bmatrix} -4 & 0 & 4 \\ 0 & 6 & 0 \\ 4 & 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ x \end{bmatrix} = -6 \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ $\Rightarrow -4x + 4z = -6x \Rightarrow x + 2z = 0$ $6y = -6y \Rightarrow y = 0$ $[4x + 2z = -6z \Rightarrow x + 2z = 0]$ $\mathbf{v}_3 = \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} \text{ (or equivalent)}$	M1 A1F A1cao	3	Substitute 'their value of a ' and attempt to get a system of equations. Both equations "correct" FT their a
(d)	$\mathbf{D} = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & -6 \end{bmatrix}$ $\mathbf{U} = \begin{bmatrix} 0 & 1 & -2 \\ 1 & 0 & 0 \\ 0 & 2 & 1 \end{bmatrix}$	B1F M1 A1cao	3	Diagonal matrix using -6 and "their 4" and "their 6" FT their non-zero \mathbf{v}_3 in \mathbf{U} \mathbf{U} correct and corresponding to \mathbf{D}
	Total		11	

Q	Solution	Marks	Total	Comments
	<p>Alternative to 7(c)</p> $\begin{bmatrix} -4 & 0 & 4 \\ 0 & 6 & 0 \\ 4 & 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = -6 \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ $\Rightarrow \begin{bmatrix} 2 & 0 & 4 \\ 0 & 12 & 0 \\ 4 & 0 & 8 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ $r_3 \rightarrow r_3 - 2r_1 \quad \begin{bmatrix} 2 & 0 & 4 \\ 0 & 12 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ $\Rightarrow y = 0 \quad x = -2z$ $\mathbf{v}_3 = \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} \quad (\text{or equivalent})$	<p>(M1)</p> <p>(A1F)</p> <p>(A1cao)</p>		<p>Row operations</p> <p>“correct” FT their a</p>

Q	Solution	Marks	Total	Comments
8(a)(i)	$\overrightarrow{AB} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$ $\overrightarrow{AD} = \begin{bmatrix} 4 \\ 3 \\ -1 \end{bmatrix}$	B1	3	either \overrightarrow{AB} or \overrightarrow{AD}
	$\begin{vmatrix} \mathbf{i} & 2 & 4 \\ \mathbf{j} & -1 & 3 \\ \mathbf{k} & 3 & -1 \end{vmatrix} = \begin{pmatrix} -8 \\ 14 \\ 10 \end{pmatrix}$	M1 A1cao		one component of $\overrightarrow{AB} \times \overrightarrow{AD}$ correct all correct
(ii)	$\text{Area } ABCD = \overrightarrow{AB} \times \overrightarrow{AD} $ $= \sqrt{8^2 + 14^2 + 10^2}$ $= \sqrt{64 + 196 + 100}$ $= \sqrt{360}$ $= \sqrt{36} \sqrt{10}$ $= 6\sqrt{10}$	M1 A1cso	2	FT their $ \overrightarrow{AB} \times \overrightarrow{AD} $ or $p = 6$
(b)	$\overrightarrow{AB} \times \overrightarrow{AD}$ is perpendicular to plane $ABCD$ Hence direction ratios of line = $-8:14:10$	M1		used for \mathbf{v} in vector line equation
	$= -4:7:5$	A1		or any multiple of $\begin{bmatrix} -4 \\ 7 \\ 5 \end{bmatrix}$
	M is mid-point of either diagonal $= \left(\frac{1+7}{2}, \frac{0+2}{2}, \frac{2+4}{2} \right)$ $= (4, 1, 3)$	B1		mid-point calculation
Hence line is $\left(\mathbf{r} - \begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix} \right) \times \begin{bmatrix} -4 \\ 7 \\ 5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$	A1cso	4	All correct or equivalent multiple of $\begin{bmatrix} -4 \\ 7 \\ 5 \end{bmatrix}$	

Q	Solution	Marks	Total	Comments
8(c)(i)	<p>Perpendicular vector to $\Pi = \begin{bmatrix} -4 \\ 7 \\ 5 \end{bmatrix}$</p> <p>$\Rightarrow \Pi$ has equation $-4x + 7y + 5z = c$</p> <p>Through $(6, 5, 17)$</p> <p>$\Rightarrow c = -4(6) + 7(5) + 5(17)$</p> <p>$= -24 + 35 + 85$</p> <p>$= 96$</p> <p>Equation is $-4x + 7y + 5z = 96$</p> <p>$x = 4 - 4t; \quad y = 1 + 7t; \quad z = 3 + 5t$</p> <p>Line meets plane when</p> <p>$-4(4 - 4t) + 7(1 + 7t) + 5(3 + 5t) = 96$</p> <p>$-16 + 16t + 7 + 49t + 15 + 25t = 96$</p> <p>$90t = 90$</p> <p>$t = 1$</p> <p>$\Rightarrow$ point of intersection = $(0, 8, 8)$</p>	M1 m1 A1 B1F M1 A1cao	6	ft their perpendicular vector using $(6, 5, 17)$ ACF parametric form of line substitution of parametric form and attempt to solve for t correct point of intersection
(ii)	<p>Volume = $(\overline{AB} \times \overline{AD}) \cdot \overline{AQ}$</p> <p>$\overline{AQ} = \begin{bmatrix} 5 \\ 5 \\ 15 \end{bmatrix}$</p> <p>$\Rightarrow \begin{bmatrix} -8 \\ 14 \\ 10 \end{bmatrix} \cdot \begin{bmatrix} 5 \\ 5 \\ 15 \end{bmatrix} = -40 + 70 + 150$</p> <p>$= 180$ (cubic units)</p> <p>Alternative</p> <p>Vol = Area of base \times perp dist</p> <p>Perp distance</p> <p>$= \sqrt{(0-4)^2 + (8-1)^2 + (8-3)^2}$</p> <p>$= \sqrt{16 + 49 + 25}$</p> <p>$= \sqrt{90}$</p> <p>Volume = $6\sqrt{10} \times \sqrt{90}$</p> <p>$= 6 \times 30$</p> <p>$= 180$ (cubic units)</p>	M1 A1F A1cso (M1) (A1F) (A1cso)	3	Attempt to use formula Follow through $\overline{AB} \times \overline{AD}$ from (a)(i). May use \overline{BQ} etc instead of \overline{AQ} Volume formula used Perp distance calculated FT their points or the equation of their plane or $\frac{ -4 \times 1 + 7 \times 0 + 5 \times 2 - 96 }{\sqrt{(-4)^2 + 7^2 + 5^2}} = \sqrt{90}$ etc
	Total		18	
	TOTAL		75	

Q	Solution	Marks	Total	Comments
	<p>Alternative to 8(c)(i)</p> $\mathbf{r} = \begin{bmatrix} 6 \\ 5 \\ 17 \end{bmatrix} + s \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} + t \begin{bmatrix} 4 \\ 3 \\ -1 \end{bmatrix}$ $\begin{cases} i & 2+2s+4t & -4 \\ j & 4-s+3t & 7 \\ k & 14+3s-t & 5 \end{cases} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ $-26s + 22t = 78$ $22s + 16t = -66$ $10s + 40t = -30$ $s = -3, t = 0$ $\begin{bmatrix} 0 \\ 8 \\ 8 \end{bmatrix}$ <p>Alternative 2 to 8(c)(i)</p> $\mathbf{r} = \begin{bmatrix} 6 \\ 5 \\ 17 \end{bmatrix} + s \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} + t \begin{bmatrix} 4 \\ 3 \\ -1 \end{bmatrix}$ $\mathbf{r} = \begin{pmatrix} 4-4p \\ 1+7p \\ 3+5p \end{pmatrix}$ $2s + 4t + 4p = -2$ $-s + 3t - 7p = -4$ $3s - t - 5p = -14$ $p = 2$ $t = 0 \text{ and } s = -3$ $\begin{bmatrix} 0 \\ 8 \\ 8 \end{bmatrix}$	<p>(M1)</p> <p>(m1) (B1F)</p> <p>(A1)</p> <p>(M1)</p> <p>(A1)</p> <p>(M1)</p> <p>(B1F)</p> <p>(m1)</p> <p>(M1)</p> <p>(A1)</p> <p>(A1)</p>		<p>$\mathbf{r} = \mathbf{a} + s\mathbf{d}_1 + t\mathbf{d}_2$ fully correct</p> <p>Substitute in $(\mathbf{r} - \mathbf{a}) \times \mathbf{b} = \mathbf{0}$ Use of $\mathbf{r} - \mathbf{a}$ in parametric form-simplified</p> <p>Three correct equations obtained from vector product-terms collected</p> <p>Correctly solving equations to get both parameters</p> <p>Correct point of intersection</p> <p>$\mathbf{r} = \mathbf{a} + s\mathbf{d}_1 + t\mathbf{d}_2$ fully correct</p> <p>Parametric form of line</p> <p>Equating components, simplifying and attempting to solve-must at least reduce to 2 equations in two unknowns</p> <p>Solving equations-one parameter correct All values correct</p> <p>Correct point of intersection</p>